fluid. Common terrestrial evaporite minerals are halite, gypsum and anhydrite, which can form as sea water evaporates, and the rocks limestone and dolostone. On Titan the composition would probably be a hydrocarbon layer from fully or partially dissolved and undissolved organic material that entered a liquid methane/ethane/nitrogen body from an accumulation of sedimenting organic haze or geothermal sources. If the evaporite material were porous it could contain a significant amount of subsurface liquid that would be available to replenish methane lost to photo-dissociation.

A second possible mechanism involves volcanic extrusion of liquid water/ammonia which ponded to form a very flat surface and then froze. This is less likely to be the mechanism producing the Arecibo echoes, however, because the radar cross-sections are more consistent with organic material. This objection may be overcome by a surface deposit of organic sediment, a specific example of our next proposal.

A third possibility is that deposits of organic haze over geologic timescales have filled in topographically low areas with a flat top facilitated by liquid methane/ethane/nitrogen that later evaporated, or by wind action. These deposits may well be porous and contain significant amounts of subsurface liquid methane/ethane/nitrogen that now buffers the atmosphere.

Although this possibility and the other mechanisms we propose may succeed in accounting for a solid surface smooth enough to produce a specular signature at 13 cm, we point out that any solid surface will roughen at 13-cm scales over geologic time owing to a variety of geologic or aeolian (wind-related) processes, and so we may need to invoke a geologically young age as well as one of these mechanisms to explain the surfaces we see.

In addition, current large-scale cloud activity on Titan is rare outside the southern high latitudes but is predicted to move north as Titan's southern summer turns to southern autumn[10], suggesting the possibility that these regions are seasonally refreshed with precipitation, which could work to smooth the surface. Unless there is a subsurface reservoir capable of sustaining atmospheric methane, the long-term prognosis is for a Titan atmosphere depleted in methane, having profound implications for photochemistry, radiative balance and circulation.

Received 10 February; accepted 18 May 2005.

1. Flasar, F. M. Oceans on Titan. *Science* **221**, 55–57 (1983).
2. Lunine, J. I., Stevenson, D. J. & Yung, Y. L. Ethane ocean on Titan. *Science* **222**, 1229–1230 (1983).
3. Yung, Y. L., Allen, M. & Pinto, J. P. Photochemistry of the atmosphere of Titan: Comparison between model and observations. *Astrophys. J. Suppl.* **55**, 465–506 (1984).
4. Campbell, D. B., Black, G. J., Carter, L. M. & Ostro, S. J. Radar evidence for liquid surfaces on Titan. *Science* **302**, 431–434 (2003).
5. Porco, C. C. *et al.* Imaging Titan from the Cassini Spacecraft. *Nature* **434**, 159–168 (2005).
6. Evans, K. F. & Stephens, G. L. A new polarized radiative transfer model. *J. Quant. Spectrosc. Radiat. Trans.* **46**, 413–423 (1991).
7. Deuze, J. L., Herman, M. & Santer, R. Fourier series expansion of the transfer equation in the atmosphere-ocean system. *J. Quant. Spectrosc. Radiat. Trans.* **41**, 483–494 (1989).
8. Brown, R. H. *et al.* The Cassini Visual and Infrared Mapping Spectrometer (VIMS) investigation. *Space Sci. Rev.* **115**, 111–168 (2004).
9. West, R. A. & Smith, P. H. Evidence for aggregate particles in the atmospheres of Titan and Jupiter. *Icarus* **90**, 330–333 (1991).
10. Brown, M. E., Bouchez, A. H. & Griffith, C. A. Direct detection of variable tropospheric clouds near Titan's south pole. *Nature* **420**, 795–797 (2002).
11. Bouchez, A. H., Brown, M. E., Schaller, E. L. & Roe, H. G. Cloud frequency and wind speed in Titan's troposphere. *Bull. Am. Astron. Soc.* **36**, 1696 (2004).
12. Wizinowich, P. *et al.* Performance of the W.M. Keck observatory natural guide star adaptive optic facility: the first year at the telescope. *Proc. SPIE* **4007**, 2–13 (2000).
13. Gibbard, S. G. *et al.* Titan's 2 μm surface albedo and haze optical depth in 1996–2004. *Geophys. Res. Lett.* **32**, L17502 (2004).
14. Lemmon, M. T. *Properties of Titan's Haze and Surface.* PhD thesis, Arizona Univ. (1994).
15. Grieger, B., Lemmon, M. T., Markiewicz, W. J. & Keller, H. U. Inverse radiation modeling of Titan's atmosphere to assimilate solar aureole imager data of the Huygens probe. *Planet. Space Sci.* **51**, 147–158 (2003).
16. Khare, B. N. *et al.* Optical constants of organic tholins produced in a simulated Titanian atmosphere—from soft-x-ray to microwave-frequencies. *Icarus* **60**, 127–137 (1984).
17. Martonchik, J. V. & Orton, G. S. Optical constants of liquid and solid methane. *Appl. Opt.* **33**, 8306–8317 (1994).

Acknowledgements We benefited from radiative transfer benchmark calculations provided by H. Gordon and from discussions with D. Campbell, R. Lorenz and S. Ostro. Part of this work was performed by the Jet Propulsion Laboratory, California Institute of Technology under contract with the National Aeronautics and Space Administration.

Author Contributions M.E.B., A.H.B. and H.G.R. supplied the Keck observations and data analysis. S.V.S. provided the light-scattering model calculations.

Author Information Reprints and permissions information is available at npg.nature.com/reprintsandpermissions. The authors declare no competing financial interests. Correspondence and requests for materials should be addressed to R.A.W. (Robert.A.West@jpl.nasa.gov).

LETTERS

Partial quantum information

Michał Horodecki[1], Jonathan Oppenheim[2] & Andreas Winter[3]

Information—be it classical[1] or quantum[2]—is measured by the amount of communication needed to convey it. In the classical case, if the receiver has some prior information about the messages being conveyed, less communication is needed[3]. Here we explore the concept of prior quantum information: given an unknown quantum state distributed over two systems, we determine how much quantum communication is needed to transfer the full state to one system. This communication measures the partial information one system needs, conditioned on its prior information. We find that it is given by the conditional entropy—a quantity that was known previously, but lacked an operational meaning. In the classical case, partial information must always be positive, but we find that in the quantum world this physical quantity can be negative. If the partial information is positive, its sender needs to communicate this number of quantum bits to the receiver; if it is negative, then sender and receiver instead gain the corresponding potential for future quantum communication. We introduce a protocol that we term 'quantum state merging' which optimally transfers partial information. We show how it enables a systematic understanding of quantum network theory, and discuss several important applications including distributed compression, noiseless coding with side information, multiple access channels and assisted entanglement distillation.

In any given situation, we can ask: how much is there to know? And, how large is our ignorance? The formulation of these questions addresses the quantity of information, not its content, which is simply because the latter is hard to assess and to compare. The former approach to classical information was pioneered by Shannon[1], who provided the tools and concepts to answer the first of the two questions: the amount of information originating from a source is the memory required to faithfully represent its output. In the case discussed in this work—of a statistical source—this amount is given by its entropy.

To approach the second question, let us consider a two-player game. One participant (Bob) has some incomplete prior information Y, the other (Alice) holds some information X: we think of X and Y as random variables, and Bob has prior information owing to possible correlations between X and Y. If Bob wants to learn X, how much additional information does Alice need to send him? This is one of the key problems of classical information theory, because it describes a ubiquitous scenario in information networks. It was solved by Slepian and Wolf[3], who proved that the amount of information that Bob needs is given by a quantity called the 'conditional entropy'. It measures the partial information that Alice must send to Bob so that he gains full knowledge of X given his previous knowledge from Y, and it is just the difference between the entropy of (X, Y) taken together (the total information) and the entropy of Y (the prior information). This result is remarkable because Alice communicates without knowing what Bob already knows: she only needs to know her distribution, and the value of the conditional entropy. Of course, this partial information is always a positive quantity. Classically, there would be no meaning to negative information.

In the quantum world, the first of the two questions, how to quantify quantum information, was answered by Schumacher[2], who showed that the minimum number of quantum bits required to convey quantum information is given by the quantum (von Neumann) entropy. To answer the second question, we now consider the quantum version of the two-party scenario above: Alice and Bob each possess a system in some unknown quantum state with the total density operator being ρ_{AB} and each party having states with density operators ρ_A and ρ_B respectively. In the case where Bob is correlated with Alice, he has some prior information about her state. We now ask how much additional quantum information Alice needs to send him, so that he has the full state (with density operator ρ_{AB}). Because we want to quantify the quantum partial information, we are interested in the minimum amount of quantum communication to do this, allowing unlimited classical communication—the latter type of information being far easier to transmit than the former, as it can be sent over a telephone, whereas the former is extremely delicate and must be sent using a special quantum channel.

We are interested in information theoretic (asymptotic) quantities, in the spirit of Shannon, so we go to the limit of many copies of state ρ_{AB} and vanishing but non-zero errors in the protocol. We find here that the amount of partial quantum information that Alice needs to send Bob is given by the quantum conditional entropy, which is exactly the same quantity as in the classical case but with the Shannon entropy changed to the von Neumann entropy:

$$S(A|B) = S(AB) - S(B) \quad (1)$$

where $S(B)$ is the entropy of Bob's state ρ_B and $S(AB)$ is the entropy of the joint state ρ_{AB}. For quantum states, the conditional entropy can be negative[4–6], and thus it is rather surprising that this quantity has a physical interpretation in terms of how much quantum communication is needed to gain complete quantum information: that is, possession of a system in the total state ρ_{AB}.

However, in the above scenario, the negative conditional entropy can be clearly interpreted. We find that when $S(A|B)$ is negative, Bob can obtain the full state using only classical communication, and additionally, Alice and Bob will have the potential to transfer additional quantum information in the future at no further cost to them. They end up sharing $-S(A|B)$ Einstein–Podolsky–Rosen (EPR) pairs[7], that is, pure maximally entangled states $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, which can be used to teleport[8] quantum states between the two parties using only classical communication. Negative partial information thus also gives Bob the potential to receive future quantum information for free. The conditional entropy plays the same role in quantum information theory as it does in the classical theory, except that here, the quantum conditional entropy can be negative in an operationally meaningful way. We can say that the ignorance of Bob—the conditional entropy—if it is negative, precisely cancels the amount by which he knows too much[9]; the negative conditional entropy is just the potential future communication gained.

This solves the well-known puzzle of how to interpret the quantum conditional entropy, which has persisted despite interesting attempts

[1]Institute of Theoretical Physics and Astrophysics, University of Gdańsk, 80-952 Gdańsk, Poland. [2]Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, UK. [3]Department of Mathematics, University of Bristol, Bristol BS8 1TW, UK.

to understand it[6]. Because there are no conditional probabilities for quantum states, $S(A|B)$ is not an entropy as in the classical case. But by going back to the definition of information in terms of storage space needed to hold a message or state, we can make operational sense of this quantity.

We now turn to the protocol which allows Alice to transfer her state to Bob's site in the above scenario (we henceforth adopt the common usage of referring directly to manipulations on a 'state'—meaning a manipulation on a physical system in some quantum state). We call this transfer 'quantum state merging', because Alice is effectively merging her state with that of Bob's. We note that in quantum information theory, faithful state transmission means that while the state merging protocol may depend on the density operator of the source, it must succeed with high probability for any pure state sent. An equivalent and elegant way of expressing this criterion is to imagine that ρ_{AB} is part of a pure state $|\psi\rangle_{ABR}$, which includes a reference system R. Alice's goal is to transfer the state ρ_A to Bob, and we demand that after the protocol, the total state still has high fidelity with $|\psi\rangle_{ABR}$ (meaning they are nearly identical); see Fig. 1 which includes a high-level description of the protocol. The essential element of state merging is that ρ_R must be unchanged, and Alice must decouple her state from R. This also means (seemingly paradoxically) that as far as any outside party is concerned, neither the classical nor quantum communication is coupled with the merged state.

Let us consider three simple and instructive examples:

(1) Alice has a completely unknown state which we can represent as the maximally mixed density matrix $\rho_A = \frac{1}{2}(|0\rangle\langle 0|_A + |1\rangle\langle 1|_A)$, and Bob has no state (or a known state $|0\rangle_B$). In this case, $S(A|B) = 1$ and Alice must send one qubit down the quantum channel to transfer her state to Bob. She could also send half of an EPR state to Bob, and use quantum teleportation[8] to transfer her state.

(2) The classically correlated state $\rho_{AB} = \frac{1}{2}(|00\rangle\langle 00|_{AB} + |11\rangle\langle 11|_{AB})$. We imagine this state to be part of a pure state with the reference system R, $|\psi\rangle_{ABR} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_R + |1\rangle_A|1\rangle_B|1\rangle_R)$. In this case, $S(A|B) = 0$, and thus no quantum information needs to be sent. Indeed, Alice can measure her state in the basis $|0\rangle \pm |1\rangle$, and inform Bob of the result. Depending on the outcome of the measurement, Bob and R will share one of two states $|\phi^{\pm}\rangle_{BR} = \frac{1}{\sqrt{2}}(|0\rangle_B|0\rangle_R \pm |1\rangle_B|1\rangle_R)$, and by a local operation, Bob can always transform the state to $|\psi\rangle_{A'BR} = \frac{1}{\sqrt{2}}(|0\rangle_{A'}|0\rangle_B|0\rangle_R + |1\rangle_{A'}|1\rangle_B|1\rangle_R)$ with A' being an ancilla at Bob's site. Alice has thus managed to send her state to Bob, while fully preserving their entanglement with R.

(3) For the state $|\phi^{+}\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$, $S(A|B) = -1$, and Alice and Bob can keep this shared EPR pair to allow future transmission of quantum information, whereas Bob creates the EPR pair $|\phi^{+}\rangle_{A'B}$ locally. That is, transferring a pure state is trivial because the pure state is known and can be created locally.

We now make a couple of observations about state merging. First, the amount of classical communication that is required is given by the number of quantum codes (see description of Fig. 1) in Alice's projection: the quantum mutual information $I(A:R) = S(A) + S(R) - S(AR)$ between Alice and the reference R. Secondly, the measurement of Alice makes her state completely factorized from R, thus reinforcing the interpretation of quantum mutual information as the minimum entropy production of any local decorrelating process[10,11]. This same quantity is also equal to the amount of irreversibility of a cyclic process: Bob initially has a state, then gives Alice her share (communicating $S(A)$), which is finally merged back to him (communicating $S(A|B)$). The total quantum communication of this cycle is $I(A:R)$ quantum bits.

Because state merging is such a basic primitive, it allows us to solve a number of other problems in quantum information theory fairly easily. We now sketch four particularly striking applications.

Distributed quantum compression: for a single party, a source emitting states with density matrix ρ_A can be compressed at a rate given by the entropy $S(A)$ of the source by performing quantum data compression[2]. Let us now consider the distributed scenario—we imagine that the source emits states with density matrix $\rho_{A_1A_2...A_m}$, and distributes it over m parties. The parties wish to compress their shares as much as possible so that the full state can be reconstructed by a single decoder. Until now, the general solution of this problem has appeared intractable[12], but it becomes very simple once we allow classical side information for free, and use state merging.

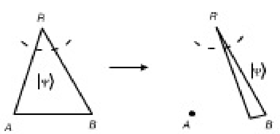

Figure 1 | Diagrammatic representation of the process of state merging. Initially, the state $|\psi\rangle$ is shared between the three systems: R(eference), A(lice) and B(ob). After the communication, Alice's system is in a pure state, while Bob holds not only his but also her initial share. Note that the reference's state ρ_R has not changed, as indicated by the curve separating R from AB. The protocol for state merging is as follows: Let Alice and Bob have a large number n of the state ρ_{AB}. To begin, we note that we only need to describe the protocol for negative $S(A|B)$, as otherwise Alice and Bob can share $nS(A|B)$ EPR pairs (by sending this number of quantum bits) and create a state $|\psi\rangle_{AA'BB'R}$ with $S(AA'|BB') < 0$. This is because adding an EPR pair reduces the conditional entropy by one unit. However, $S(A|B) < 0$ is equivalently expressed as $S(A) > S(AB) = S(R)$, and in this case (compare refs 16–18) measurement in a uniformly random basis on Alice's n systems projects Bob and R into a state $|\phi\rangle_{BR}$ whose reduction to R is very close to ρ_R. But this means that Bob can, by a local operation, transform $|\phi\rangle_{BR}$ to $|\psi\rangle_{ABR}$. Finally, by coarse-graining the random measurement, Alice essentially projects onto a good quantum code[16–18] of rate $-S(A|B)$; this still results in Bob obtaining the full state ρ_{AB}, but now, just under $-nS(A|B)$ EPR pairs are also created. These codes can also be obtained by an alternative construction[12].

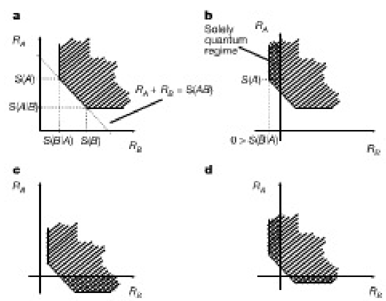

Figure 2 | The rate region for distributed compression by two parties with individual rates R_A and R_B. The total rate R_{AB} is bounded by $S(AB)$. **a**, The rate region of a source with positive conditional entropies; **b** and **c**, the purely quantum case of sources where $S(B|A) < 0$ or $S(A|B) < 0$. It is even possible that both $S(B|A)$ and $S(A|B)$ are negative, as shown in **d**, but observe that the rate-sum $S(AB)$ must be positive. If one party compresses at a rate $S(B)$, then the other party can over-compress at a rate $S(A|B)$, by merging her state with the state that will end up with the decoder. Time-sharing gives the full rate region, since the bounds evidently cannot be improved. Analogously, for m parties A_k, and all subsets $T \subseteq \{1, 2, ..., m\}$ holding a combined state with the entropy $S(T)$, the rate sums $R_T = \sum_{k\in T} R_k$ have to obey $R_T > S(T|\bar{T})$ with $\bar{T} = \{1, 2, ..., m\}\setminus T$ the complement of set T.

Remarkably, the parties can compress the state at the total rate $S(A_1A_2...A_m)$—the Schumacher limit[2] for collective compression—even though they must operate separately. This is analogous to the classical result: the Slepian–Wolf theorem[3]. We describe the quantum solution for two parties and depict the rate region in Fig. 2.

A closely related problem to distributed compression is noiseless coding with side information. Here, only Alice's state needs to arrive at the decoder, while Bob can send part of his state to the decoder in order to help Alice lower her rate. The classical case of this problem was introduced by Wyner[13]. For the quantum case, we require that the full state ρ_{AB} is preserved in the protocol, but do not place any restriction on what part of Bob's state may be at the decoder and what part can remain with him. For one-way protocols, we find by using state merging that if ρ_A and ρ_B are encoded at rates R_A and R_B respectively, then the decoder can recover ρ_A if and only if $R_A \geq S(A|U)$ and $R_B \geq E_p(AU:R) - S(A|U)$. Here R is the purifying reference system, U and V are systems with their joint state produced by acting with some unitary on B and $E_p(AU:R) = \min_\Lambda S(AU\Lambda(V))$ is the entanglement of purification[14]. The minimum is taken over all channels Λ acting on V.

In addition to the two central questions of information theory we asked earlier—*how much is there to know? how large is our ignorance?*—information theory also concerns itself with communication rates. In the quantum world, the rate at which quantum information can be sent down a noisy channel is related to the coherent information $I(A\rangle B)$ which was previously defined as[15] $\max\{S(B) - S(AB), 0\}$. This quantity is the quantum counterpart of Shannon's mutual information; when $I(A\rangle B)$ is maximized over input states, it gives the rate at which quantum information can be sent from Alice to Bob via a noisy quantum channel[16–18]. As with the classical conditional entropy, Shannon's classical mutual information[1] is always positive, and indeed it makes no sense to have classical channels with negative capacity. However, the relationship between the coherent information and the quantum channel capacity contained a puzzle. The channel capacity was thought to be meaningful as a positive quantity. So the coherent information was defined as the maximum of 0 and $S(B) - S(AB)$, since it could be negative.

We will see that negative values do make sense, and thus propose that $I(A\rangle B)$ should not be defined as above, but rather as $I(A\rangle B) = -S(A|B)$. It turns out that negative rate, although impossible in classical information theory, has its interpretation in a situation with two senders.

We imagine that Alice and Bob wish to send independent quantum states to a single decoder Charlie via a noisy channel which acts on both inputs. This problem is considered in ref. 19. Our approach using state merging also provides a solution when either of the coherent informations are negative, and gives the following larger achievable rates, compared to ref. 19:

$$R_A \leq I(A\rangle CB), \quad R_B \leq I(B\rangle CA), \quad R_A + R_B \leq I(AB\rangle C) \qquad (2)$$

where R_A and R_B are the rates of Alice and Bob for sending quantum states. Here, we use our redefinition of the coherent information, in which we allow it to be negative. In achieving these rates, Alice can send (or invest) $I(A\rangle C)$ quantum bits to merge her state with the decoder. The second sender (Bob) then already has Alice's state at the decoder, and can send at the higher rate $I(B\rangle AC)$. This provides an interpretation of negative achievable rates: if a channel of one sender has negative coherent information, this means that she has to invest this amount of entanglement to help her partner achieve the highest rate. The protocol is for one of the senders to merge his state with the state held by the decoder. The expressions in (2) are in formal analogy with the classical multiple access channel.

Entanglement of assistance: consider Alice, Bob and $m - 2$ other parties sharing (many copies of) a pure quantum state. The entanglement of assistance E_A (ref. 20) is defined as the maximum entanglement that the other parties can create between Alice and Bob by local measurements and classical communication. This was recently solved for up to four parties[21], and can be generalized to an arbitrary number m of parties using state merging (using universal codes that depend only on the density matrix of the helper, as described in Fig. 1). We find that the maximal amount of entanglement that can be distilled between Alice and Bob, with the help of the other parties, is given by the minimum entanglement across any bipartite cut of the system which separates Alice from Bob:

$$E_A = \min_T \{S(AT), S(B\bar{T})\} \qquad (3)$$

where the minimum is taken over all possible partitions of the other parties into groups T and its complement $\bar{T} = \{1, ..., m-2\}\setminus T$. To achieve this, each party in turn merges their state with the remaining parties, preserving the minimum cut entanglement.

We have described a fundamental quantum information primitive—state merging—and demonstrated some of its many applications. There are also conceptual implications. For example, the celebrated strong subadditivity of quantum entropy[22], $S(A|BC) \leq S(A|B)$, receives a clear interpretation and transparent proof: having more prior information makes state merging cheaper. Our results also shed new light on the foundations of quantum mechanics: it has long been known that there are no conditional probabilities, so defining conditional entropy is problematic. Merely replacing classical entropy with quantum entropy gives a quantity which can be positive or negative. Quite paradoxically, only the negative part was understood operationally—as quantum channel capacity—which made the problem even more obscure. State merging 'annihilates' these problems with each other. It turns out that the puzzling form of quantum capacity as a conditional entropy is the flip side of our interpretation of quantum conditional entropy as partial quantum information, which makes equal sense in the positive and negative regime. The key point is to realize that in the negative regime, we can gain entanglement and transfer Alice's partial state, while in the positive regime, only the partial state is transferred.

Remarkably, and despite the formal analogy, the classical scenario does not occur as a classical limit of the quantum scenario—we consider both classical and quantum communication, and there is no meaning to preserving entanglement in the classical case. We have only just begun to grasp the full implications of state merging and negative partial information; a longer technical account (M.H., J.O. and A.W., manuscript in preparation), with rigorous proofs and further application will appear elsewhere.

Received 20 February; accepted 9 June 2005.

1. Shannon, C. E. A mathematical theory of communication. *Bell Syst. Tech. J.* 27, 379–423, 623–656 (1948).
2. Schumacher, B. Quantum coding. *Phys. Rev. A* 51, 2738–2747 (1995).
3. Slepian, D. & Wolf, J. K. Noiseless coding of correlated information sources. *IEEE Trans. Inf. Theory* 19, 461–480 (1971).
4. Wehrl, A. General properties of entropy. *Rev. Mod. Phys.* 50, 221–260 (1978).
5. Horodecki, R. & Horodecki, P. Quantum redundancies and local realism. *Phys. Lett. A* 194, 147–152 (1994).
6. Cerf, N. J. & Adami, C. Negative entropy and information in quantum mechanics. *Phys. Rev. Lett.* 79, 5194–5197 (1997).
7. Einstein, A., Podolsky, B. & Rosen, N. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* 47, 777–780 (1935).
8. Bennett, C. H. *et al.* Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* 70, 1895–1899 (1993).
9. Schrödinger, E. Die gegenwärtige Situation der Quantenmechanik. *Naturwissenschaften* 23, 807–821, 823–828, 844–849 (1935).
10. Groisman, B., Popescu, S. & Winter, A. On the quantum, classical and total amount of correlations in a quantum state. Preprint at ⟨http://arXiv.org/quant-ph/0410091⟩ (2004).
11. Horodecki, M. *et al.* Local versus non-local information in quantum information theory: formalism and phenomena. *Phys. Rev. A* 71, 062307 (2005).
12. Ahn, C., Doherty, A., Hayden, P. & Winter, A. On the distributed compression of quantum information. Preprint at ⟨http://arXiv.org/quant-ph/0403042⟩ (2004).
13. Wyner, A. On source coding with the side information at the decoder. *IEEE Trans. Inf. Theory* 21, 294–300 (1975).
14. Terhal, B. M., Horodecki, M., DiVincenzo, D. P. & Leung, D. The entanglement